

5.4 – Applications of Exponential and Power Equations

Daily Objectives:

1. Find solutions to real-world applications of rational-exponential, exponential, and power functions:

Example 1: Rita wants to deposit \$500 into a savings account so that its doubling time will be 8 years. What annual percentage rate is necessary for this to happen? (Assume the interest on the account is compounded annually).

$$\begin{aligned}
 y &= ab^x \\
 \frac{1000}{500} &= \frac{500b^8}{500} \\
 2 &= b^8 \\
 2^{\frac{1}{8}} &= b \\
 1.0905 &= b
 \end{aligned}
 \qquad
 r = 9.05\%$$

Example 2: Maria is working with a chemical compound that has a half-life of 20 days. Find the decay rate of Maria's compound if she starts with 50 grams of her compound.

$$\begin{aligned}
 25 &= 50b^{20} \\
 \frac{1}{2} &= b^{20} \\
 \left(\frac{1}{2}\right)^{\frac{1}{20}} &= b \\
 .965936 &= b
 \end{aligned}
 \qquad
 \begin{aligned}
 \text{decay rate} &= 1 - .965936 \\
 &= .034064 \\
 &= 3.41\%
 \end{aligned}$$

Example 3: Sandy is growing bacteria cells for an experiment. After 2 hours, there are 10 bacteria cells. After 5 hours, Sandy has 25 bacteria cells. Sandy is wondering how fast her bacteria are growing. Find an exponential equation to model Sandy's data.

$$\begin{aligned}
 &(2, 10) \quad (5, 25) \\
 \begin{aligned}
 y &= ab^x \\
 y &= ab^x \\
 \frac{25}{5} &= \frac{ab^5}{ab^2} \\
 5 &= b^3 \\
 5^{\frac{1}{3}} &= b \\
 1.709976 &= b
 \end{aligned}
 \qquad
 \begin{aligned}
 y &= ab^x \\
 10 &= a(1.709976)^2 \\
 \frac{10}{1.709976^2} &= \frac{10}{2.924} \\
 3.4199519 &= a \\
 y &= 3.41995(1.70998)^x
 \end{aligned}
 \end{aligned}$$

Review:

1. Solve each equation for positive values of x . If answers are not exact, approximate to the nearest *hundredth*.

a. $\sqrt[3]{x} = 2.6$

$x = 2.6^3$
 $x = 17.576$ (Exact)

b. $x^{1/4} = 0.2$

$x = .2^4$
 $x = .0016$ (Exact)

c. $0.75x^5 - 8 = -3$

$\frac{.75x^5}{.75} = \frac{5}{.75}$
 $x^5 = 6.\bar{6}$
 $x = (6.\bar{6})^{1/5}$
 $x = 1.46$

d. $4(x^{5/6} + 7) = 159$

$\frac{4}{4}x^{5/6} + \frac{28}{4} = \frac{159}{4}$
 $x^{5/6} + 7 = 39\frac{3}{4}$
 $x^{5/6} = 32\frac{3}{4}$
 $x = (32\frac{3}{4})^{6/5}$
 $x = 65.80$

2. Rewrite each expression in the form ax^n .

a. $(8x^9)^{2/3}$

$(\sqrt[3]{8})^2 x^{\frac{18}{3}}$
 $2^2 x^3$
 $4x^3$

b. $(81x^{12})^{3/4}$

$(\sqrt[4]{81})^3 x^{\frac{36}{4}}$
 $3^3 x^9$
 $27x^9$

c. $(49x^{10})^{1/2}$

$\sqrt{49} x^5$
 $7x^5$